TEST ASSIGNMENTS

1. Given some concrete (e.g. 3×3 inhomogeneous system of linear equations), something consisting of three (perhaps 4) equations in the three unkowns (x, y, z) check whether this equation is solvable (by setting up the matrix and applying Gauss elimination). If the system has solutions try to find the "general solution" (specific solution plus general solution to the homogeneous system);

2. Consider the linear equation:

$$
\begin{aligned}
4x + 5y + z &= 11 \\
5x + 3y + 3z &= 9 \\
x + 5z &= 5
\end{aligned}
$$

   What is the solution to this system of linear equations? Is it uniquely determined?

3. Create a system matrix of rank 2, e.g. `C = rand(3,2) * rand(2,3)`. Then the *homogeneous system* `C*z=v` has non-trivial (i.e. non-zero) solutions (for $v \neq 0$). Determine the rank of the matrix C, which should be 2 in almost all cases. Consequently Gauss elemination should give you two non-trivial rows (as a control), and the Null-space is $3 - 2 = 1-$dimensional, resp. `nC = null(C)` should give one single vector (of length $=$ norm 1). Hence for any solvable ($=$ consistent) solution of $C * z_0 + v$ the general solution of the linear system is given as $z_0 + \lambda \cdot nC, \lambda \in \mathbf{R}$.

4. Plot the function $y \mapsto y^3 \cdot sin(y)$ on $[0, 2pi]$ and find a numerical approximation to the position of its minimal value on this interval (if possible with precision of at least 2 or 3 digits). Give the figure a title and save the figure if you have time (for your orientation: the minimum should be approx. -124 and it is taken near 5.2).

5. Given two vectors in $\mathbf{R}^n$, do the first step of Gram-Schmidt "numerically" (by yourself, step by step, and NOT just by applying the ORTH command (which you can/should use for checking the correctness of your result); I suggest to take $H = zeros(3, 2); H(:) = 1 : 6$ as an example.

6. Start with `subplot(2,1,1)`. Plot the function $cos(3 * x)$ over $[0, 2 * pi]$ (you may use as a bases $x = 0 : 0.01 : 2*pi$ or $x = linspace(0, 2*pi, 301)$ for the plot. Then do `subplot(2,1,2)` and plot $cos(3*2*pi*x)$ over the interval $[0, 1]$ for comparison. From this you may conclude that MATLAB's cosine function uses arc-length (and not degrees), and hence the same is true for the inverse function (called acos(y)).

7. Verify experimentally that the rank of a product of a random 3×5 - matrix with another random 5×3 (any order) is a matrix of rank 3. Do this by computing 500 (or you choose a number) of such random products, and indicate the number of failures (if there are any) within this experiment.

8. Let us fabricate a random symmetric matrix in the following way: `n = 5; O = orth(rand(n)); D = diag(rand(1,n))`, `A = O * D * O'`, , then obviously $A == A'$. Starting from a random vector $x = rand(n, 1)$ iterate in the following way: x = A * x; x = x/norm(x); home; disp(x); Repeating this, say 10 or 100 times shows that the vector x is getting stable after a while (it is the direction of the largest eigenvector, that comes out via the so-called vector iteration). One can also start the whole game with $O = eye(n)$, i.e. by taking $A = diag(rand(n))$; See what the result is and try to give (from the experiments) an explanation!!

9. Given a finite sequence **c** describing a polynomial $p(x) \in \mathcal{P}_r(\mathbf{R})$, it can be split into linear factors of the form $\prod_{k=1}^{r}(x - \lambda_k)$, where the sequence $(\lambda_k)_{k=1}^{r}$ is the sequence of ROOTS of that polynomial. Those complex numbers can be easily obtained numerically (they are complex numbers in general, even if the polynomial is real) by the command `rts = roots(c);`.

   Verify (experimentally) that this is a true statement, i.e. choose a sequence of (say 4) random points in $[0, 1]$ and verify (using the `polyval`-command) that $p(x)$ in effect equals (up to numerical precision) such a product.

10. Fabricate a so-called LAGRANGE interpolating polynomial, i.e. given a number of points (for simplicity on the real axis), say $(\lambda_k)_{k=1}^{K}$, for some $K > 3$ (to make it interesting). Determine a polynomial $L_3(x)$ of degree K such that $L_3(\lambda_3) = 1$ while at the same time $L_3(\lambda_k) = 0$ for an $k \neq 3$. Plot the result!

11. Describe the polynomial $p(x) = x + x^2 + \cdots x^6$ in matlab and plot its real values over the unit circle (say over the unit roots of order 360, to make it concrete.

12. Generate a random orthonormal system $O5$ for R^5, choose tow random vector $h1, h2 \in \mathbf{R}^5$ and show that the angle between them does not change if one applies the mapping $x \mapsto O5 * x$. In particular,, the norms of those two vectors remain unchanged under this application. Recall that

$$cos(\alpha) = \langle h_1, h_2 \rangle / (norm(h1) \cdot norm(h2))$$

13. .

14. .

15. .

1. TEST